## Chapter 2: Mechanical Equilibrium

## $\rightarrow$ <br> Weight of a Sign

## Purpose

To resolve a force vector into horizontal and vertical components

## Required Equipment/Supplies

2 spring balances
strong cord or heavy string
ring stand or vertical rod with table clamp
$5 / 8$-inch dowel, 0.4 m long with screw eyehook
1-kg mass
graph paper
protractor

## Discussion

Let's consider the forces that act on a sign on the side of a building. Look at where the cable connects with the boom on the sign. When we label all the forces acting on a body, it's referred to as a free-body diagram. Whereas a sketch of the problem helps us visualize the problem, a freebody diagram helps us analyze the forces that act on the body. Since the sign is at equilibrium, the net force on the sign is zero. This means the magnitude of the forces on the sign upward equal those downward, and the magnitude of the forces to the left equal those to the right. Now let's draw a free-body diagram and label forces acting on the sign at that point.



Fig. B

Remember to label the forces acting on the sign-not the forces the sign exerts on something else. First, there is gravity, which acts down on the sign, or weight. The second is the cable pulling up at an angle. The third is the boom pushing sideways (away from the wall). To help visualize the boom force, consider what would happen if only the weight and the cable force acted on the sign - the sign would fall down and go (boom!) right into the wall!

Figure A illustrates a boom arrangement similar to the one we've discussed. It consists of a cable connected to a firm support and tied to a horizontal rod or boom at its other end. The load or weight is suspended from the junction of the cable and the boom represents the weight of the sign. In this apparatus a dowel rod with a screw eye serves as the boom.

The upper spring balance measures the tension in the cord used as a cable. The tension in the cable can be resolved into components. Its vertical component supports the load or weight and its horizontal component pushes against the boom, as shown in the vector diagram of Figure A, compressing it. Whereas tension stretches an object, compression squishes or presses inwards on it.

In this experiment, you will predict the value of the horizontal and vertical components of the cable tension by constructing a vector diagram similar to the one you've just drawn, using the measured values for the force and a protractor to measure the angle. You will then compare these predicted values with the values you obtain by direct measurement.

## Procedure

Step 1: As a warm-up activity, let's resolve a force vector into horizontal and vertical components. One method is the so-called parallelogram method, where the resultant vector is the diagonal of two sides or components of a parallelogram. We use arrows to represent vectors and draw them from the tail, or beginning of the vector, to the end, or the tip of the vector. The tip of the arrow points in the direction of the vector and the length represents the magnitude when drawn to scale.

First, to represent the force as a vector as shown in Figure B, choose a scale that will give a large diagram (that fills most of the graph paper). Then, draw a line in the direction of the force at $37^{\circ}$ to the horizontal, making its length proportional to the magnitude of the force and placing an arrowhead at its end. Here the vector $c$ represents a force of 5 newtons. Now, draw a horizontal and a vertical (dashed) line at the tail of this vector so that the vector $c$ is the hypotenuse of a right triangle. Label the legs of the triangle $b$ and $a$, and determine their magnitude using the scale. Here, $b$ is 4 newtons and $a$ is 3 newtons.

Step 2: Set up the boom as in Figure A so that it is horizontal and the cord makes any convenient angle with the boom. Use a protractor to measure angle $A$, the angle between the boom and the cord. Record its value in Table A. Read the spring balance connected to the cord and record its value for $c$ in Table A.

Step 3: Choose a scale and draw the vector $c$ on a sheet of paper. Draw a right triangle around the vector $c$ as indicated above and determine the value of $b$ and $a$, the horizontal and vertical components, from the diagram. Record these in Table A as the predicted values.

Step 4: Measure the horizontal component $b$ of the tension by connecting a second spring balance to the screw eye on the boom as shown in Figure A. Pull slowly in the horizontal direction until the boom just falls away from the vertical support rod. Record the spring balance reading at this instant. This is the measured value of the component of $b$. Record the magnitude of $b$ in Table A.

Step 5: Now measure the vertical component of the tension, vector $a$. Again attach the spring balance to the screw eye and pull upward (vertically) until the spring balance completely supports the weight $W$ and the boom. The reading of the balance is the measured value of the vertical component $a$. Record it in the table.

Step 6: Compare the predicted value of the components with the measured value in each case. Repeat the experiment using a different angle for the cable and a different weight $W$.

## Data Table A

| Trial | Given <br> Force $c$ <br> $(\mathrm{~N})$ | Angle $A$ <br> $\left({ }^{\circ}\right)$ | Predicted <br> Value of $a$ <br> $(\mathrm{~N})$ | Predicted <br> Value of $b$ <br> $(\mathrm{~N})$ | Measured <br> Value of $a$ <br> $(\mathrm{~N})$ | Measured <br> Value of $b$ <br> $(\mathrm{~N})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |

## Analysis

1. Under what circumstances is the tension in the cable equal to the load?
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$\qquad$
2. If angle $A$ is made smaller, what will happen to the tension in the cable? Why?
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$\qquad$
$\qquad$
3. How was the weight of the boom taken into account in this experiment?

## Going Further

Set up the derrick arrangement shown in Figure C. Design an experiment to discover (a) how the tension in the cable varies as angle $A$ increases from $20^{\circ}$ to $60^{\circ}$ and (b) how the force of the boom varies as angle $A$ increases from $20^{\circ}$ to $60^{\circ}$.


Fig. C

