Name Period Date

15 Impact Speed

Purpose

To estimate the speed of a falling object as it strikes the ground

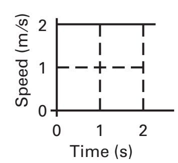
Required Equipment/Supplies

stopwatch
string with a rock tied to one end
object with a large drag coefficient, such as a leaf or feather
plastic foam ball or table-tennis ball

Discussion

The area of a rectangle is its height multiplied by its base. The area of a triangle is its height multiplied by half its base. If the height and base are measured in meters, the area is measured in square meters. Consider the area under a graph of speed vs. time. The height represents the speed measured in meters per second, m/s, and the base represents time measured in seconds, s. The area of this patch is the speed times the time, expressed in units of m/s times s, which equals m. The speed times the time is the distance traveled. *The area under a graph of speed vs. time represents the distance traveled*. This very powerful idea underlies the advanced mathematics called integral calculus. You will investigate the idea that the area under a graph of speed vs. time can be used to predict the behavior of objects falling in the presence of air resistance.

d = area under "curve" of graph
$= \triangle v \triangle t$
= (2 m/s) (2s) = 4 m



If there were no air friction, a falling table-tennis ball or plastic foam ball would fall at a constant acceleration g so that its change of speed would be

$$v_f - v_i = gt$$

where v_f = final speed

v_i = initial speed

g = acceleration of gravity

t = time of fall

A graph of speed vs. time of fall is shown in Figure A, where $v_i = 0$. The y-axis represents the speed v_f of the freely falling object at the end of any time t. The area under the graph line is a triangle of base t and height v_f, so the area equals $\frac{1}{2}v_f t$.

To check that this does equal the distance traveled, note the following. The *average* speed $\bar{v}$ is half of v_f. The distance d traveled by a con-

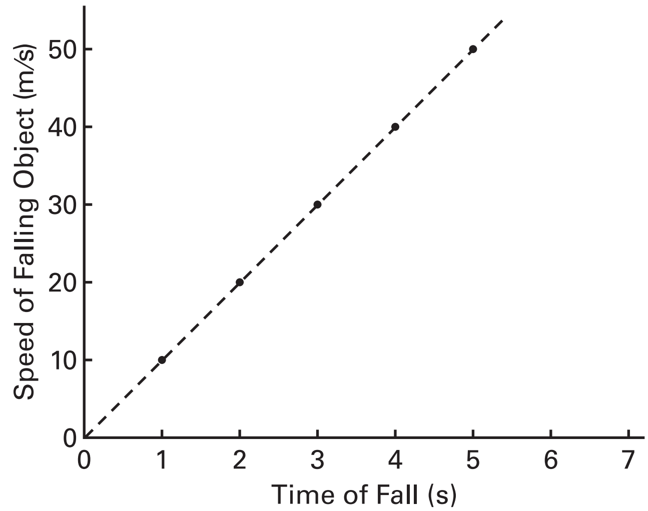


Fig. A

stantly accelerating body is its average speed $\bar{v}$ multiplied by the duration t of travel.

$$d = \bar{v}t \text{ or } d = \frac{1}{2}v_f t$$

If you time a table-tennis ball falling from rest a distance of 43.0 m in air (say, from the twelfth floor of a building), the fall takes 3.5 s, *longer* than the theoretical time of 2.96 s. Air friction is *not negligible* for most objects, including table-tennis balls. A graph of the actual speed vs. the time of fall looks like the curve in Figure B.

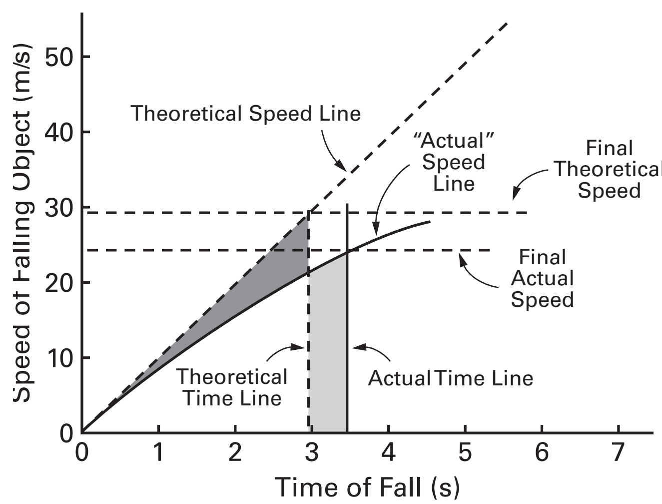


Fig. B

Since air resistance reduces the acceleration to below the theoretical value of 9.8 m/s^2, the falling speed is less than the theoretical speed. The difference is small at first, but grows as air resistance becomes greater and greater with the increasing speed. The graph of actual speed vs. time curves away from the theoretical straight line.

Is there a way to sketch the actual speed-vs.-time curve from only the distance fallen and the time of fall? Calculate the theoretical time of fall for no air resistance. The *height* from which the object is dropped is the same with or without air resistance. *The area under the actual speed-vs.-time curve must be the same as the area under the theoretical speed-vs.-time line.* It is the distance fallen. On a graph of theoretical speed vs. time, draw a vertical line from the theoretical time of fall on the horizontal axis up to the theoretical speed-vs.-time line. (In Figure B, this line is labeled "theoretical time line.") Draw another vertical line upward from the *actual* time of fall on the horizontal axis. (This line is labeled "actual time line" in Figure B.) Sketch a curve of actual speed vs. time that crosses the second vertical line below the theoretical speed-vs.-time line. Sketch this curve so that the area added below it due to increased time of fall (stippled area) equals the area subtracted from below the theoretical speed-vs.-time line due to decreased speed (cross-hatched area). *The areas under the two graphs are then equal.* This is a fair approximation to the actual speed-vs.-time curve. The point where this curve crosses the vertical line of the actual time gives the probable impact speed of the table-tennis ball.

Procedure

Step 1: Your group should choose a strategy to drop a table-tennis ball or plastic foam ball and clock its time of fall within 0.1 s or better. Consider a long-fall drop site, various releasing techniques, and reaction times associated with the timer you use.

Devise dropping strategy.

Step 2: Devise a method that eliminates as much error as possible to measure the distance the object falls.

Measure falling height.

Step 3: Submit your plan to your teacher for approval.

Submit proposal to teacher.

Step 4: Measure the height and the falling times for your object using the approved methods of Steps 1 and 2.

Measure height and falling times.

height = ______________________

actual time of fall = ______________

Step 5: Using your measured value for the height, calculate the theoretical time of fall for your ball. Remember, this is the time it would take the ball to reach the ground if there were no air resistance.

Compute theoretical falling time.

theoretical time = ________________

Step 6: Using an overhead transparency or graph paper, trace Figure B, leaving out the actual speed-vs.-time curve. Draw one vertical line from the *theoretical* time of fall for your height up to the theoretical speed-vs.-time line. Draw the other vertical line from the *actual* time of fall up to the theoretical speed-vs.-time line.

Draw theoretical and actual speed lines.

Step 7: Starting from the origin, sketch your approximation for the actual speed-vs.-time curve out to the point where it crosses the actual time line, using the example mentioned in the discussion. The area of your stippled region should be the same as that of the cross-hatched region.

Sketch curve on graph.

One possible way to do this is to tape a piece of cardboard to the wall and project your transparency onto it. Trace your two regions onto the cardboard and cut them out. Then measure the mass of the two regions. If their masses are not the same, adjust your actual speed curve and try again. Your approximation is done when the two regions have the same mass.

Step 8: Draw a horizontal line from the upper right corner of your stippled region over to the speed axis. Where it intersects the speed axis is the object's probable impact speed.

Predict probable impact speed.

Analysis

1. Have your teacher overlap your graph with those of others. How does your actual speed-vs.-time curve compare with theirs?

2. What can you say about objects whose speed-vs.-time curves are close to the theoretical speed-vs.-time line?

3. What does the area under your speed-vs.-time graph represent?

4. The equation for distance traveled is $d = \bar{v}t$. In this lab, the distance fallen is the same with or without air friction. How do the average speeds and times compare with and without air friction? Try to use different-sized symbols such as those used on page 66 of your text.

5. If you dropped a large leaf from the Empire State Building, what would its speed-vs.-time graph look like? How might it differ from that of a baseball?

6. The *terminal* speed of a falling object is the speed at which it stops accelerating. How could you tell whether an object had reached its terminal speed by glancing at an actual speed-vs.-time graph?